

Neutrino Speed and Temperature

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Abstract

Dependence on the temperature of the integration constants of the relativistic quantum Hamilton-Jacobi leads to Lorentz symmetry breaking, a consequence of the nonlocal nature of the quantum potential. There is an intriguing analogy with the spontaneous symmetry breaking of the Lorentz group in the context of thermal QFT and KMS states, a basic issue in noncommutative geometry. Interestingly, also in thermal QFT the breaking of symmetries, including Lorentz symmetry, occur even in cases with no interactions. Here we consider the correction term $\sqrt{mc^2/\pi K T} e^{-\frac{mc^2}{KT}}$. Using the estimate value of $20K/km$ for the Earth’s crust thermal gradient, one gets $m_{\nu_\mu} \approx 0.46 \text{ eV}$ that reconciles OPERA, MINOS and SN1987A data and explains why superluminality is not observed in other experiments. We also show that the observed deviation of the speed of light $(c_\gamma - c)/c \approx 1.7 \times 10^{-11}$ would correspond to a photon mass $m_\gamma \approx 2.3 \times 10^{-23} \text{ eV}$. This would provide a natural explanation of the apparently paradoxical consequences of violating Lorentz symmetry. Finally, we derive the Lagrangian leading to the above expressions for the velocity and show that the change of the relativistic equations is simply described by means of a thermal coordinate.

A possible explanation of OPERA's results [1] has been recently proposed in [2]. There it has been observed the curious coincidence between OPERA's results and the kinematics defined by the relativistic quantum Hamilton-Jacobi equation, derived by first principles in [3, 4, 5]. Such a formulation is strictly related to classical-quantum duality [6].

Consider the stationary Klein-Gordon equation $(-\hbar^2 c^2 \Delta + m^2 c^4 - E^2)\psi = 0$. In one-dimensional space the associated quantum Hamilton-Jacobi equation is

$$(\partial_q S_0)^2 + m^2 c^2 - \frac{E^2}{c^2} + \frac{\hbar^2}{2} \{S_0, q\} = 0, \quad (1)$$

where $\{f, q\} = \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2$ denotes the Schwarzian derivative of f . The term $Q = \frac{\hbar^2}{4m} \{S_0, q\}$, is the quantum potential and $p = \partial_q S_0$ the conjugate momentum. It is crucial that Q , which is always non-trivial [3, 4, 5], is quite different with respect to the one by de Broglie and Bohm. In [2] it has been shown that, upon averaging the mean speed on the period $[q, q + \frac{\pi}{k}]$, $k = \sqrt{E^2 - m^2 c^4} / \hbar c$, one gets $v = c \frac{\sqrt{E^2 - m^2 c^4}}{E} / L_1$ with $L_1 \leq 1$ an integration constant of (1). Dependence of the initial conditions of (1) on the temperature T leads to contributions to the dispersion relation depending on T . We will show that

$$v = c \frac{\sqrt{E^2 - m^2 c^4}}{E} (1 + \sqrt{mc^2 / \pi K T} e^{-\frac{mc^2}{KT}}),$$

reconciles OPERA, MINOS and SN1987A, giving $m_{\nu_\mu} \approx 0.46$ eV and $v = c$ for massless particles, if any, and explains why the Cohen-Glashow argument [8] does not apply to the above expression for v . Furthermore, since the next lightest known massive particle, the electron, is approximately 10^3 heavier than m_ν , by $e^{-\frac{m_e}{KT}} = (e^{-\frac{m_{\nu_\mu}}{KT}})^{\frac{m_e}{m_{\nu_\mu}}}$, one sees that to have measurable effects on the electron one would need a temperature $T \approx 10^5 K$, so explaining why corrections with respect to experimental data on the Earth have not been observed. Also note that the maximal speed is $v_{Max} = c + c / \sqrt{2\pi e}$, which holds only for massive particles and is reached at $T = 2mc^2 / K$. Another remarkable fact is that the observed deviation of the speed of light $(c_\gamma - c) / c \approx 1.7 \times 10^{-11}$ [7] would correspond to a photon mass $m_\gamma \approx 2.3 \times 10^{-23}$ eV. Even an extremely thin value of the mass photon would provide a natural explanation of the apparently paradoxical consequences of violating Lorentz symmetry.

Since the quantum potential plays the central rôle in our construction, it is worth stressing that the derivation and formulation of the quantum version of the Hamilton-Jacobi equations of [3, 4] is quite different from the one by de Broglie-Bohm. In particular, the quantum potential is basically different. Let us consider the case of non-relativistic quantum mechanics. de Broglie-Bohm made the identification $\psi = R e^{\frac{i}{\hbar} S_0}$ with ψ the wave-function, not just a general complex solution of the Schrödinger equation, as it should be on mathematical grounds [3, 4]. This leads to the following trouble. Consider the case in which the wave-function is the eigenfunction of the Hamiltonian corresponding to a one-dimensional bound state, *e.g.* the harmonic oscillator. In this case ψ is proportional to a real function. It follows that S_0 is a constant, so that the conjugate momentum is trivial. This would imply that at the quantum level the particle is at rest and starts moving in the classical limit. This is a well-known paradox observed by Einstein. In the derivation considered in [3, 4] such a paradox is naturally resolved by construction: $\{S_0, q\}$

is well-defined if $\partial_q \mathcal{S}_0$ is never vanishing. In particular, the general solution of the non-relativistic quantum stationary Hamilton-Jacobi equation, which is formally equivalent to (1), with $(m^2 c^4 - E^2)/2mc^2$ replaced by $V - E$, is

$$e^{\frac{2i}{\hbar} \mathcal{S}_0 \{\delta\}} = e^{i\alpha} \frac{w + i\bar{\ell}}{w - i\ell}, \quad (2)$$

with $w = \psi^D/\psi \in \mathbb{R}$, where now ψ and ψ^D are two real linearly independent solutions of the stationary Schrödinger equation. $\delta = \{\alpha, \ell\}$, $\alpha \in \mathbb{R}$ and $\ell = \ell_1 + i\ell_2$ are integration constants. Note that $\ell_1 \neq 0$ even when $E = 0$, equivalent to having $\mathcal{S}_0 \neq \text{const}$, which is a necessary condition to define $\{\mathcal{S}_0, q\}$. This implies a non-trivial \mathcal{S}_0 , even for a particle classically at rest. One has $\psi = \mathcal{S}_0'^{-1/2} (Ae^{\frac{i}{\hbar} \mathcal{S}_0} + Be^{-\frac{i}{\hbar} \mathcal{S}_0})$, and $\psi \in \mathbb{R}$ implies $|A| = |B|$, so that \mathcal{S}_0 is non-trivial. The $\hbar \rightarrow 0$ limit is subtle and leads to the appearance of fundamental constants [3, 4, 5]. A basic observation in [2] is that averaging the period of the oscillating terms, which also appear in the non-relativistic case, leads to \hbar -independent solutions that, besides including the standard one, describe other solutions depending on the values of the integration constants (see also later). We note that the above decomposition of ψ , now called bipolar decomposition, is successfully used in studying molecular trajectories [9, 10] (see also [11]).

Another distinguished feature of the formulation introduced in [3, 4] is that energy quantization follows without any axiomatic interpretation of the wave-function. As a result, the quantum potential, intrinsically different from the one by de Broglie-Bohm, is always non-trivial. Like mc^2 , it plays the rôle of intrinsic energy and it is at the basis of the quantum behavior. For example, besides energy quantization, it makes transparent its rôle in the tunnel's effect, where guarantees that the conjugate momentum always takes real values.

The general solution of (1) is (2), where now ψ and ψ^D are two real linearly independent solutions of the Klein-Gordon equation. Following Floyd [13], time parametrization is defined by Jacobi's theorem $t = \frac{\partial \mathcal{S}_0}{\partial E}$. Since $p = \partial_q \mathcal{S}_0$, it gives the group velocity $v = \partial E / \partial p$. Set $L = k\ell$, $L_1 = \Re L$, $L_2 = \Im L$. The mean speed is

$$v = \frac{q}{t} = c \frac{\sqrt{E^2 - m^2 c^4}}{E} \frac{|\sin(kq) - iL \cos(kq)|^2}{L_1},$$

that for $L = 1$ reduces to the classical relativistic relation $v = c \frac{\sqrt{E^2 - m^2 c^4}}{E}$. In the following we will consider the case $|L| = 1$, so that

$$v = c \frac{\sqrt{E^2 - m^2 c^4}}{E} \frac{1 + L_2 \sin(2kq)}{L_1}. \quad (3)$$

The integration constants L_1 and L_2 may depend on particle's quantum numbers, energy and fundamental constants as well. As we said, here we will consider an intriguing dependence of L on the temperature. Such a possibility is related to a new way of considering the $\hbar \rightarrow 0$ limit which has been introduced in [2]. Let us first note that because of the \hbar^{-1} term in $\sin(2kq)$, that typically is very strongly oscillating, the $\hbar \rightarrow 0$ limit is not well-defined. One possibility, considered in [3, 4], is that the integration constants may depend on E , \hbar and other fundamental constants. In this way the term \hbar^{-1} is cured by

a suitable dependence on \hbar of L , a procedure that leads to consider the Planck length (note that ℓ has the dimension of a length). The new way of getting the classical limit considered in [2] is based on the averaging of the oscillations. This is reminiscent of the Dirac's averaging of the oscillating part of the free electron's speed $\frac{i}{2}\hbar\dot{\alpha}_1^0 e^{-2iHt/\hbar} H^{-1}$ (see Dirac's treatment of the free electron in his book). In [2] it has been observed that in the case of the OPERA's experiment, the term $\sin(2kq)$ is very rapidly oscillating, with the neutrino Compton wavelength being approximately 10^{11} times k^{-1} . This is a stringent physical motivation to average v on the interval $[q, q + \frac{\pi}{k}]$. By (3) (here we denote $\langle v \rangle$ of [2] by v)

$$v = \frac{k}{\pi} \int_q^{q+\frac{\pi}{k}} v(q') dq' = c \frac{\sqrt{E^2 - m^2 c^4}}{E} \frac{1}{L_1}. \quad (4)$$

In [2] it has been noted that this may indicate that neutrinos live on a discrete space-time.

The basic outcome is that now the classical solution depends on L_1 , whose value, by construction, can be also less than one. This leads to deviations of the standard classical dispersion relation. Now note that a possible dependence of L on the temperature should be not a surprise. Actually, as observed by Dirac in computing the speed of the free electron, the uncertainty principle implies that it does not make sense considering scales which are much shorter than the Compton wavelength. On other hand, averaging on the period may lead to the breaking of the Lorentz group. This is just reflected by possible values of L_1 which are less than the unity. Furthermore, it is natural to expect that such an averaging leads to a dependence on the degrees of freedom of the averaged space domain, which in general is not the empty space. In doing this we should remind the analogy with spontaneous symmetry breaking of Lorentz group in thermal QFT, we will shortly discuss later. Also note that the quantum potential is strictly related to the Fisher information and Shannon entropy [12] (see also [11]). It is then natural to expect that also here the temperature plays the central rôle. For such reasons, further discussed below, we will consider the case in which L depends on T .

Remarkably, dependence of L on the temperature is allowed just as a consequence of a basic physical property such as linearity of quantum mechanics. In the case of the Klein-Gordon equation, as in the case of the Schrödinger equation, such a property is equivalent to the invariance of the Schwarzian derivative under Möbius transformations. In this respect, note that a basic identity at the basis of [3, 4, 5] is that $(\partial_q \mathcal{S}_0)^2$ can be expressed as the difference of two Schwarzian derivatives $\left(\frac{\partial \mathcal{S}_0}{\partial q}\right)^2 = \frac{\beta^2}{2} \left(\{e^{\frac{2i}{\beta} \mathcal{S}_0}, q\} - \{\mathcal{S}_0, q\} \right)$, that forces us introducing the dimensional constant β , that is \hbar . We also note the appearance of the imaginary factor. It follows that Eq.(1) is equivalent to the Schwarzian equation $\{e^{\frac{2i}{\hbar} \mathcal{S}_0}, q\} = \frac{2}{\hbar^2 c^2} (E^2 - m^2 c^4)$, which is invariant under a Möbius transformation of $e^{\frac{2i}{\hbar} \mathcal{S}_0}$ so that we can also assume that α and ℓ depend on the temperature so that Eq.(2) has the form

$$e^{\frac{2i}{\hbar} \mathcal{S}_0 \{\delta(T)\}} = e^{i\alpha(T)} \frac{w + i\bar{\ell}(T)}{w - i\ell(T)}. \quad (5)$$

This provides an intriguing possibility in the case of non-relativistic quantum mechanics. Actually, possible dependence on the temperature provides a sort of dynamics non detected by the wave-function and may open a new view on its collapse. As the temperature changes, $e^{\frac{2i}{\hbar} \mathcal{S}_0 \{\delta(T)\}}$ moves on the boundary of the Poincaré disk (discrete jumps by Möbius transformations are also allowed) with $\{e^{\frac{2i}{\hbar} \mathcal{S}_0}, q\}$ remaining invariant. This happens also

when $\{e^{\frac{2i}{\hbar}S_0}, q\}$ is considered in the case of the relativistic quantum Hamilton-Jacobi equation (1). In particular, $\{e^{\frac{2i}{\hbar}S_0}, q\} = \frac{2}{\hbar^2 c^2}(E^2 - m^2 c^4)$ is invariant under variations of both the coordinate and the temperature.

Eq.(5) may be related to $t - T$ duality and non-commutative geometry (see [14] and references therein). In this respect, it is worth recalling de Broglie view [15]. In particular, in the framework of relativistic thermodynamics, he considered $t_m = h/mc^2$ as a particle internal time and identified it with an internal temperature $KT_m = mc^2$. de Broglie also suggested that, in the case of non-relativistic quantum mechanics, the collapse of the wave-function is related to a sort of particle's Brownian interaction with the environment.

Time-temperature duality is a basic feature of QFT, related to the analogy with classical statistical mechanics where the inverse temperature plays the rôle of imaginary time. Time and temperature also mix in performing analytic continuation along complex paths in the path-integral. Another well-known analogy concerns the transition amplitude for a particle for the time it that coincides with the classical partition function for a string of length t at $\beta = 1/\hbar$ [16]. It is widely believed that such a dualities are deeply related to the properties of space-time and should emerge in the string context.

In [14] it has been considered a string model where time-temperature and classical-quantum dualities, emerge naturally. It turns out that the solitonic sector of compactified strings have a dual description as quantum statistical partition function on higher dimensional spaces, built in terms of the Jacobian torus of the string worldsheet and of the compactified space. More precisely, in [14] it has been shown that in the case of compactification on a circle

$$\sum_{m,n \in \mathbb{Z}^g} e^{-\beta S_{m,n}} = \text{tr } e^{-\beta H} , \quad (6)$$

where $\beta = 2R^2/\alpha'$, with R the compactification radius, and the Hamiltonian H is $\Delta_{J_\Omega}/2\pi$, with Δ_{J_Ω} the Laplacian on the Jacobian torus J_Ω of the worldsheet. Eq.(6) is just the direct consequence of the stronger identity $H\Psi_{m,n} = S_{m,n}\Psi_{m,n}$, that is the set $\{S_{m,n}|m,n \in \mathbb{Z}^g\}$ coincides with the spectrum of H , a result deeply related to the theory of Riemann surfaces [17].

Temperature-time duality naturally emerges as a consequence of the complexified version of T -duality, a fundamental feature of string theory. By (6) the standard T -duality corresponds to the invariance, up to a multiplicative term given by powers of β , of the partition function under inversion of the temperature $\beta \rightarrow \frac{1}{\beta}$. Complexification of β has basic motivations which interplay between physics and geometry. Set $\omega(A) = \text{tr } (Ae^{-\beta H})/\text{tr } e^{-\beta H}$. Using the invariance of the trace under cyclic permutations we have

$$\omega(A(t)B) = \omega(BA(t + i\beta)) . \quad (7)$$

In such a context the complexification of β naturally appears in globally conformal invariant QFT [18]. It is worth noticing that both in [18] and in the BC system [19] the KMS (Kubo-Martin-Schwinger) states [20] play a central rôle. In particular, in the limit of 0-temperature the KMS states may be used to define the concept of point in noncommutative space, a basic issue in string theory [21].

A nice feature of the possible temperature dependence of the integration constants of (1) is that it does not arise as a consequence of a specific interaction, rather, as shown

above, it is just due to the linearity of the Klein-Gordon equation, or, equivalently, of the Möbius symmetry of the Schwarzian derivative.

The above remarks suggest asking whether there exists an $L(T)$ such that the resulting expression for the velocity agrees with the experimental data. Let us then go back to the KMS states and note that the Fourier transformed form of (7) [22]

$$FT[\omega(AB)](p) \equiv \int d^d x e^{ip(x-y)} \omega(A(x)B(y)) = e^{-\beta p_0} FT[\omega(BA)](-p) , \quad (8)$$

shows that $FT[\omega(AB)](p)$ and $e^{-\beta p_0} FT[\omega(BA)](-p)$ cannot be both Lorentz invariant. Spontaneous breakdown of Lorentz boost symmetry at $T \neq 0$ is nicely seen considering the $(1, 1)$ component of the 2×2 matrix propagator of a scalar field that at the tree level is [22]

$$FT[\omega(\varphi\varphi)](p) = \frac{1}{p^2 - m^2 + i\epsilon} + 2\pi\delta(p^2 - m^2) \frac{e^{-\beta|p_0|}}{1 - e^{-\beta|p_0|}} ,$$

which breaks Lorentz $\omega([M_{01}, T\varphi(x)\varphi(y)]) \neq 0$.

Even if we are considering a breaking of the Lorentz group from the quantum potential, which is a highly nonlocal term, it is clear that the analogy with thermal QFT suggests considering in our approach $e^{-\frac{mc^2}{KT}} = e^{-\frac{T_m}{T}}$, as order parameter of the Lorentz symmetry breaking. The crucial property of the KMS states is (7), or equivalently (8), showing that time evolution is invariant, upon commutation, under an imaginary shift of the time which is inversely proportional to the temperature. In the case of the quantum Hamilton-Jacobi equation, time is generated by applying Jacobi's theorem. Then one gets t as a function of q and a shift of time corresponds to a shift of q . On the other hand, this corresponds to a jump of $e^{\frac{2i}{\hbar}S_0}$ on the boundary of the Poincaré disk, and may be compensated by changing $L(T)$ that, depending on its functional structure, may correspond to a change of T .

In the present formulation the dependence on $e^{-\frac{mc^2}{KT}}$ as order parameter of the breaking of Lorentz symmetry comes out as an effect due to the quantum potential and it is not seen as due to some particle interaction. For such a reason it is not a surprise that a so weakly interacting particle, such as the neutrino, may get thermal contributions to the dispersion relation. Remarkably, even in this case there is an analogy with the thermal breakdown of symmetries in thermal field theory since the spontaneous breakdown of symmetries occurs even in cases with no interactions.

We then look for an expression of L_1 leading to a speed which is in agreement with the known data. We now show that there is a remarkable solution that passes several tests. Namely, by (4) it follows that choosing $1/L_1(T) = 1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}}$, leads to

$$v = c \frac{\sqrt{E^2 - m^2 c^4}}{E} \left(1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}} \right) . \quad (9)$$

Note that, just as in the case of the temperature in the Jüttner distribution, which is the one measured by an observer at the rest with the gas as a whole, even here the temperature is the one measured by an observer at the rest with the medium. Let us show that (9) passes the known possible tests. First, note that knowing the temperature distribution

along neutrino's path γ one may compute its flight time

$$\Delta t = \int_{\gamma} \frac{ds}{v(T(s))} . \quad (10)$$

Estimating the Earth's crust thermal gradient to be $20K/km$ (see below), and the room temperature $293K$ at CERN and CNGS, we have $T(s)/K = 20(r - \sqrt{s^2 - ls + r^2})10^{-3} + 293$, where $r = (l^2 + 4h^2)/8h$, with $l = 732km$ the baseline length and $h = 11.4km$ the maximal deep, we get that $(v - c)/c \approx 2.37 \times 10^{-5}$ corresponds to the neutrino mass

$$m_{\nu_{\mu}} \approx 0.46 \text{ eV} ,$$

which is in the range of its estimated value. Eq.(9) also agrees with MINOS's results [23]. Another important test is that in the case of neutrinos from the SN1987A supernova [24] one has $T \approx 0$ during the flight from the supernova to the Earth, so that

$$\sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}} \approx 0 ,$$

that is for the neutrinos coming from the SN1987A supernova Eq.(9) just provides $v \approx c$, as observed. Another crucial test for Eq.(9) is that it gives $v = c$ for massless particles, if any. Furthermore, it fixes the maximal speed to be $v_{Max} = c + c/\sqrt{2\pi e}$, which is independent of the mass but holds only for massive particles. It is reached at $T_{Max} = 2mc^2/K$, which is twice de Broglie's temperature. There is another interesting test for (9). Namely, since the next lightest known massive particle, the electron, is approximately 10^3 heavier than m_{ν} , by $e^{-\frac{mc^2}{KT}} = \left(e^{-\frac{m_{\nu\mu}c^2}{KT}}\right)^{\frac{m_e}{m_{\nu\mu}}}$, one sees that measurable effects on the electron need a temperature $T \approx 10^5 K$. This would explain why corrections with respect to experimental data on the Earth have not been observed. Also note that for $m_{\nu_{\mu}}$ the maximal speed $c + c/\sqrt{2\pi e}$ is reached at $T_{Max} = 2 \times 10^4 K$, while for the electron and proton such a velocity is reached at $T_{Max} = 10^{10} K$ and $T_{Max} = 2 \times 10^{13} K$ respectively, so that the effects are measurable only inside supernovae.

An intriguing possibility that would answer several questions on the formulation of special relativity in the case of superluminality is that massless particles do not exist. On the other hand, like a particle in S^1 , a compact Universe may imply that energy, and therefore masses, admit only discretized values with a minimal gap. A hint in such a direction is the result by Gharibyan who observed that photons with 12.7 GeV energy are moving faster than light by $5.1(1.4)mm/s$ [7]. This seems due to an energy dependence on the vacuum refraction index, which implies a temperature dependence. The measurement was done at room temperature, *i.e.* the accelerator vacuum was filled with $293K$ blackbody photons. By (9), the correction $(c_{\gamma} - c)/c \approx 1.7 \times 10^{-11}$ would correspond to a photon mass

$$m_{\gamma} \approx 2.3 \times 10^{-23} \text{ eV} ,$$

which is much lower than the upper bound 10^{-18} eV reported by the Particle Data Group.

A remarkable feature of (9) is that it also explains why the argument by Cohen and Glashow [8] for the bremsstrahlung $\nu_{\mu} \rightarrow \nu_{\mu} + e^+ + e^-$ (see also [25]) does not apply to (9). To see this let us first show that the modified relativistic relation is

$$E^2 = p^2 c^2 \left(1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}}\right)^2 + m^2 c^4 . \quad (11)$$

Replacing $(m^2c^4 - E^2)/2mc^2$ by $V - E$, (1) is formally equivalent to the non-relativistic quantum stationary Hamilton-Jacobi equation. It follows that, as observed in [3, 4], Eq.(1) can be seen as a deformation of the classical relativistic Hamilton-Jacobi equation by a “conformal factor”. Actually, noting that $\{\mathcal{S}_0, q\} = -(\partial_q \mathcal{S}_0)^2 \{q, \mathbf{s}\}$, $\mathbf{s} = \mathcal{S}_0(q)$, we see that (1) is equivalent to

$$E^2 = \left(\frac{\partial \mathcal{S}_0}{\partial q} \right)^2 c^2 [1 - \hbar^2 \mathcal{U}(\mathcal{S}_0)] + m^2 c^4, \quad (12)$$

with $\mathcal{U}(\mathcal{S}_0) = \{q, \mathbf{s}\}/2$, the canonical potential introduced in the framework of p - q duality [3, 4]. Eq.(12) can be expressed in the form $E^2 = (\partial_{\hat{q}} \mathcal{S}_0)^2 c^2 + m^2 c^4$ where $d\hat{q} = dq/\sqrt{1 - \hat{\beta}^2(q)}$ with $\hat{\beta}^2(q) = \hbar^2 \{q, \mathbf{s}\}/2$. Integrating

$$\hat{q} = \int^q \frac{dq'}{\sqrt{1 - \hat{\beta}^2(q')}}. \quad (13)$$

Eq.(13) indicates that in considering the differential structure one should take into account quantum effects on space geometry. Eq.(13) is equivalent to

$$\hat{q} = c \int^q dq' \frac{\partial_{q'} \mathcal{S}_0}{\sqrt{E^2 - m^2 c^4}} = c \int^{\mathcal{S}_0(q)} \frac{d\mathbf{s}}{\sqrt{E^2 - m^2 c^4}}. \quad (14)$$

The above investigation leads to a basic result. Namely, comparison with the dispersion relation (11) shows that the classical limit we obtained by averaging on the short period, allowing the solution with the crucial $L_1 \leq 1$, is equivalent to the thermal deformation of the coordinate

$$\lim_{\hbar \rightarrow 0} \hat{q} = q_T = \frac{q}{1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}}},$$

so that the relativistic quantum Hamilton-Jacobi equation (1) becomes

$$E^2 = \left(\frac{\partial \mathcal{S}_0}{\partial q_T} \right)^2 c^2 + m^2 c^4,$$

which is (11). Eq.(14) is clearly related to the relativistic classical action whose modification, leading to (11), is

$$L = -\frac{mc^2}{\gamma_T},$$

with

$$\gamma_T = \frac{1}{\sqrt{1 - \frac{v^2}{c^2 \left(1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}} \right)^2}}}.$$

We have

$$p = \frac{\partial L}{\partial v} = \frac{mv\gamma_T}{\left(1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}} \right)^2}, \quad (15)$$

and

$$E = pv - L = mc^2\gamma_T , \quad (16)$$

so that we get (11) and by $v = \partial_p E$,

$$v = \frac{pc^2}{E} \left(1 + \sqrt{\frac{mc^2}{\pi KT}} e^{-\frac{mc^2}{KT}} \right)^2 ,$$

which is, by (11), Eq.(9). The dispersion relation (11) is quite different from the one considered by Cohen and Glashow. In particular, here $E^2 - p^2c^2$ does not correspond to any effective mass. Another aspect in Cohen and Glashow argument is that they assume that the neutrino mass m_{ν_μ} can be neglected so that one may use the dispersion relation

$$E = vp .$$

However, as we saw, the case $m = 0$ is a critical one since (9) gives $v = c$ and (11) $E = pc$. In other words, even if very tiny, a mass different from zero is crucial to have superluminal solutions since the dispersion relations in the case $m \neq 0$ and $m = 0$ differ substantially.

A possible further test of (9) would be an experiment with the neutrino beam passing through the Earth's core whose estimated temperature is $5.700K$. Interestingly, a temperature dependence of neutrino speed would allow geophysical investigations by neutrino beams. Another check concerns possible speed fluctuations with respect neutrino's detection times at OPERA and MINOS. The point is that there is a high thermal gradient inside the Earth crust. Temperature increases by as $20 - 25K$ per kilometer in the upper part of the crust, but its gradient is smaller in deeper crust. The highest temperature crust is reached at the boundary with the underlying mantle where it ranges from about $470K$ to $670K$. Small changes of the non-inertial forces, due to the periodical motion Moon-Earth and Sun-Earth, reflect in a change of the beam trajectory in Earth's crust. This should provide a variation of the temperature distribution along neutrino's path γ and therefore of its flight time (10). Note that, due to the $e^{-\frac{mc^2}{KT}}$ term, a small change of T would provide a measurable effect on v . Also note that increasing the energy of the beam increases the local temperature so that v may have a weak dependence on the beam energy. Therefore, it would be interesting to know the thermal gradient for the first tract where neutrinos are produced and where the highest temperature is approximately $600K$. Let us also mention that, as happened in other experiments, one should also check possible effects due to tidal forces on the experimental apparatus. Finally, note that Eq.(9) may admit higher order terms, possibly energy dependent. In this respect it may be of interest considering [26].

We conclude observing that superluminal neutrinos have been considered in other works before OPERA's results, in particular in [27]. After OPERA the problem of superluminal neutrinos stimulated a lot of work (see [28] for a partial list).

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